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On the inclusive gluon jet production from the triple pomeron vertex in the perturbative QCD

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Abstract. Single and double inclusive cross-sections for gluon jet production from within the triple pomeron vertex are studied in the reggeized gluon technique in the QCD with $N_c \to \infty$. It is shown that to satisfy the AGK rules the vertex has to be fully symmetric in all four reggeized gluons which form the two final pomerons. The single inclusive cross-sections are found for different cuttings of the triple pomeron vertex. They sum to the expression obtained by Kovchegov and Tuchin in the color dipole picture. The found double inclusive cross-sections satisfy the AGK rules.

1 Introduction

In perturbative QCD at small values of x the strong interaction can be modelled by the exchange of reggeized gluons and BFKL pomerons as their bound states. In the limit of a large number of colors, $N_c \to \infty$, the model reduces to the propagation and triple interaction of pomerons in the tree diagram approximation. The equations which sum these diagrams for heavy nucleus targets ($A \gg 1$) have been written both for DIS (BK equation [1–4]) and for nucleus–nucleus collisions [5]. The solution of these equations allows one to find the total cross-sections for processes like $\gamma^* A$ and AB. The next important observables, which carry much more information about the dynamics, are inclusive cross-sections to produce gluon jets which are to hadronize into the observed hadrons. First calculations of single and double inclusive cross-sections were made in [6] on the basis of the AGK rules [7]. From them it follows in particular that in the single inclusive cross-section the produced gluon jet comes from within the initial BFKL pomeron before its branchings. Later from the dipole picture a slightly different expression for the same cross-section was derived. In it, apart from the above-mentioned naive AGK contribution, another term appeared, which could be interpreted as emission from within the triple pomeron vertex itself [8]. Such a contribution is not prohibited by the AGK rules but was usually neglected as small. However in perturbative QCD at small x it is of the same order as the emission from the pomeron. Further analysis performed in [9] in the framework of reggeized gluon diagrams seemingly discovered many terms in the contribution to production from the vertex (and among them also the one found in [8]). However the derivation in [9] was based on certain ad hoc assumptions, so that it was stressed there that the derivation was in fact quite heuristic and needed a more detailed study. The present paper, which is a direct continuation of [9], presents results of this study.

We find that a more careful analysis of reggeized gluon diagrams and especially the validity of the AGK rules for different forms in which their sums may be presented leads to results which differ from those obtained in [9]. We find that the triple pomeron contribution in the form used in that paper (with the so-called diffractive vertex Z) does not satisfy the AGK rules and only the form with the symmetric Bartels vertex V does satisfy them[1] This circumstance radically changes the derivation of the contribution to jet production from within the triple pomeron vertex. The found terms for different cuttings of the pomerons joined by the vertex are as numerous and complicated as in [9], but in the sum they indeed combine into the Kovchegov–Tuchin term. Thus the reggeized gluon digrams approach to the gluon jet production completely agrees with the dipole picture. In fact the most convenient technique combines both approaches and allows us to obtain results in the simplest way. Using it one is able to easily construct an evolution equation for the four-gluon amplitude with jet production. One can also show that contributions which blatantly violate the AGK rules found for the double inclusive cross-section in [11] are in fact absent.

2 The AGK rules

We start with the scattering amplitude of some projectile (e.g. γ^*) on two scattering centers, which corresponds to

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[1] We highly appreciate our numerous discussions with J. Bartels who has always insisted on this point (see also [10]).

the triple interaction of BFKL pomerons and is schematically shown in Fig. 1, where one can also see our notation for the initial and final momenta p_i and p'_i, $i=0,1,2$. Both the projectile and two targets are supposed to be colorless objects like quark–antiquark loops (photons or onia). In practice this amplitude, multiplied by one half of the square of the nuclear profile function $T(b)$, represents the contribution from the double rescattering in the nucleus. However, for the problem at hand this circumstance is unimportant. We assume that the c.m. energy $s=(p_0+p_1)^2=(p_0+p_2)^2$ is large and the transferred momenta $t_i=(p'_i-p_i)^2$ are finite and therefore much smaller than s. In fact in the following we shall concentrate on the forward case, $t_0=0$, which is of most practical importance. The ladders represent the initial and two final BFKL pomerons and the central blob corresponds to the triple pomeron vertex, which is local in rapidity. Allowing for the pomerons to be Regge cuts and not simple poles (as is the case of the BFKL pomeron) and for the two lower pomerons to have different energies, one gets a representation for the amplitude [12]:

$$T_{3\to 3}=\frac{1}{2\pi^4}\frac{s_1s_2}{M^2}\int\prod_{k=0}^{2}\left(\frac{\mathrm{d}j_k}{2\pi i}\zeta_k\right)\left(\frac{s_1}{M^2}\right)^{j_1-1}\left(\frac{s_2}{M^2}\right)^{j_2-1}\times\left(\frac{M^2}{Q^2}\right)^{j-1}F(j_i,t_i)\,. \quad (1)$$

The signature factors are defined as

$$\zeta=-\pi\frac{\mathrm{e}^{-\mathrm{i}\pi j}+1}{\sin(\pi j)}\,, \quad (2)$$

where for ζ_0 one should take $j=j_0-j_1-j_2$. As compared to [12] we have included a factor i for each pomeron since in the skeleton diagrams one should use elemental scattering amplitudes multiplied by i. For physical scattering we are to take $s_1=s_2=s$. The function $F(j_i,t_i)$ corresponds to the diagram of Fig. 1 in the complex angular momentum representation. It is a real function, which is a product of three pomerons in the j-representation and the triple pomeron vertex Γ:

$$F(j_i,t_i)=\sum_{a_i,b_i}\int\frac{\mathrm{d}^2q_1}{(2\pi)^2}\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\times\Gamma^{b_1b_2}_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2)P_1^{a_1a_2}(j_1;k_1,k_2)\times P_2^{a_3a_4}(j_2;k_3,k_4)P_{b_1b_2}(j_0;q_1,q_2)\,. \quad (3)$$

Here P, P_1 and P_2 are the initial and two final pomerons, a_i and b_i are their color indexes, k_i and q_i are their transverse momenta, with $q_1+q_2=p_0-p'_0$, $k_1+k_2=p_1-p'_1$ and $k_3+k_4=p_2-p'_2$. Since the pomerons are colorless, the Ps include a projector onto the colorless state:

$$P_{b_1b_2}(j_0;q_1,q_2)=\frac{1}{N_c^2-1}\delta_{b_1b_2}P(j_0;q_1,q_2) \quad (4)$$

and similarly for the final pomerons. The normalization is chosen to include an extra factor $\sqrt{N_c^2-1}$ into the function $P(q_1,p_2)$. This normalization depends on the one chosen for the external sources. In the following, to generalize for multiple scattering and relate the pomerons to the sum of fan diagrams Φ, we rescale the final pomerons as $P\to g^2P$ and the initial pomeron as $P\to P/g^2$. Then for single scattering $\Phi=\Phi^{(1)}=PT(b)$, and the vertex Γ in (3) coincides with the vertex in the BK equation.

We first demonstrate that the relation between the imaginary parts of the amplitude coming from different cuts trivially follows from the representation (1) with a real function $F(j_i,t_i)$. Note that $\operatorname{Im}\zeta=\mathrm{i}\pi$ and that for pomerons in the lowest order $\zeta=\mathrm{i}\pi$. Armed with these properties we may calculate the total and partial imaginary parts of the amplitude. Since all non-trivial dependence on energies is contained in signature factors, we have only to follow their change when taking the relevant discontinuities.

First we look at the total imaginary part. To find it we have just to substitute all ζs by $\mathrm{i}\pi$, which will result in a contribution

$$(\operatorname{Im}T)^{\mathrm{tot}}=-\frac{1}{2\pi}\frac{s_1s_2}{M^2}\int\prod_{k=0}^{2}\frac{\mathrm{d}j_k}{2\pi i}\left(\frac{s_1}{M^2}\right)^{j_1-1}\left(\frac{s_2}{M^2}\right)^{j_2-1}\times\left(\frac{M^2}{Q^2}\right)^{j-1}F(j_i,t_i)\,. \quad (5)$$

Partial imaginary parts are corresponding to different cuts. The diffractive cut corresponds to taking $\mathrm{i}\zeta_0=2\pi$ (just the discontinuity divided by i), $\xi_1=\xi_2=-\pi$ and dividing the whole expression by two. Obviously we get

$$(\operatorname{Im}T)^{\mathrm{dif}}=-(\operatorname{Im}T)^{\mathrm{tot}}\,. \quad (6)$$

The double cut corresponds to taking $\mathrm{i}\zeta_0=\mathrm{i}\zeta_1=\mathrm{i}\zeta_2=2\pi$ (again the discontinuities divided by i) and dividing the whole expression by 2×2 (2 for the imaginary part and 2

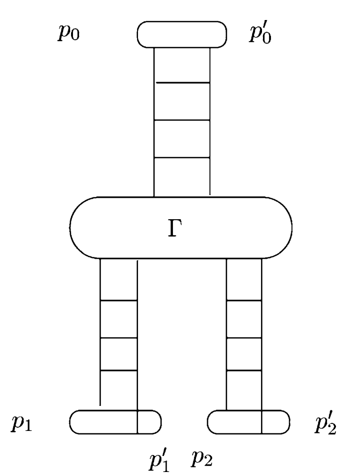

Fig. 1. Triple pomeron contribution to the $3\to 3$ amplitude

for the identity of the two legs). Obviously we get twice the diffractive cut

$$(\mathrm{Im}\,T)^{\mathrm{double}} = 2(\mathrm{Im}\,T)^{\mathrm{dif}}\,. \qquad (7)$$

Finally single cuts correspond to ξ_0 and one of $\xi_{1,2}$ substituted by 2π, the other one by $-\pi$ multiplying by 2 for the complex conjugate part and dividing by 2 to pass from the discontinuity to the imaginary part. As a result we get four times the diffractive cut with the opposite signs:

$$(\mathrm{Im}\,T)^{\mathrm{single}} = -4(\mathrm{Im}\,T)^{\mathrm{dif}}\,. \qquad (8)$$

The sum of these partial contributions is equal to the total imaginary part, and their relative weights correspond to the AGK rules.

Note however that what we have just presented is only a formal derivation of the AGK rules. In fact one has to be able to identify intermediate states and production amplitudes which generate different cuts of the amplitude in the unitarity relation for the total $\mathrm{Im}\,T$. This is trivial for the internal gluons in the pomerons themselves, but not so for the coupling of the pomerons to the external particles and to each other. Such an identification is not needed when one studies the total amplitude and the cross-section which it describes, but it becomes a necessity if one wants to see which particles are produced from the vertexes describing these couplings. The problem mostly concerns the double cut in the variables s_1 and s_2 in (1), which has to be reinterpreted as a single unitarity cut.

That this is not generally possible illustrates the diagram shown in Fig. 2, say for a scalar theory with a triple interaction. It obviously contains the double cut in lower reggeon energies, but this cut cannot be identified with the unitarity cut, which cannot pass through both reggeons. So we have to study the unitarity relation for the amplitude in correspondence with the cuts we have discussed.

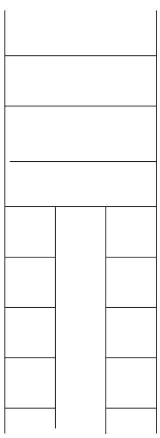

Fig. 2. Amplitude which possesses a double cut which is not unitary

Let us start from the diffractive cut, which is graphically shown in Fig. 3a, where also the corresponding intermediate states in the unitarity relation for the three participating pomerons are shown. The cut drawn through the three-pomeron vertex is purely symbolical: it is not possible to interpret the cut vertex Γ terms of intermediate states for the production amplitudes in a straightforward manner. The general formula (1) only tells us that the cut vertex is a real function independent of the cutting $\Gamma^{b_1 b_2}_{a_1 a_2 a_3 a_4}(k_1,k_2,k_3,k_4|q_1,q_2) \equiv \Gamma(1,2,3,4)$, where the numbers refer to the final gluons, 1 and 2 in the first lower pomeron and 3 and 4 in the other one. The double cut is shown in Fig. 3b together with its unitarity content. Again the cut through the vertex is not directly expressible in terms of intermediate states. One observes that it corresponds to the interchange of the final gluons 2 and 3 in the vertex: $\Gamma(1,3,2,4)$. However, as mentioned, the cut vertex should not depend on the particular cutting. It should also be symmetric in the pairs of gluons $(1,2)$ and $(3,4)$ due to the properties of the pomeron. As a result, the vertex function Γ has to be completely

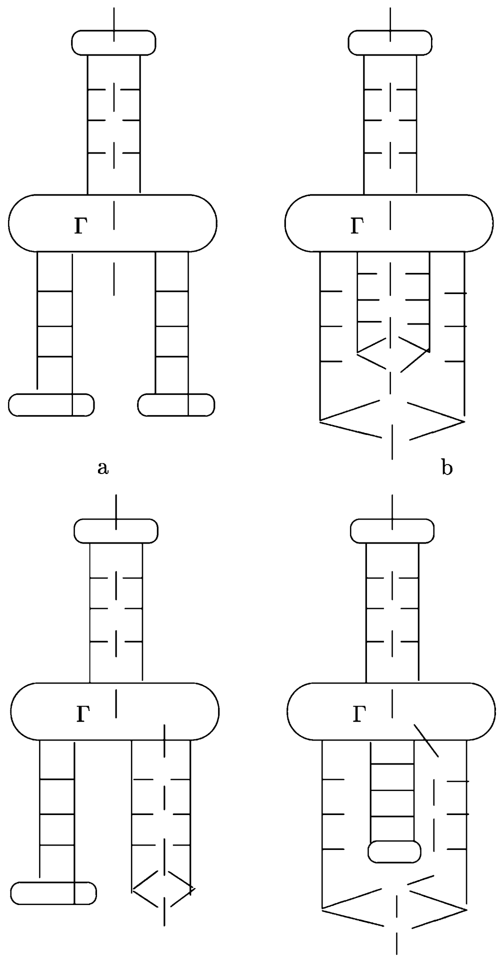

Fig. 3. a Diffractive, **b** double, and **c**,**d** single cuts and their unitarity content

symmetric in all four final gluons 1, 2, 3 and 4. The single cut contributions shown in Fig. 3c and d do not imply any new condition on the vertex function Γ. So we conclude that the necessary condition for the fulfillment of the AGK rules for the triple pomeron contribution represented according to (1) is the complete symmetry of the triple pomeron vertex in all four final reggeized gluons. This requirement generalizes the one in the original AGK derivation that the vertex should not change with different cuttings.

Note that the contributions to the total imaginary part of the amplitude $T_{3\to 3}$ can be classified not only by a particular cutting, diffractive, double or single, but also by the number of gluons emitted at the vertex rapidity. In the lowest order it can be zero or unity. So each of the contributions from a particular cutting can be split in two:

$$(\mathrm{Im}\, T)^{\mathrm{cut}} = \sum_{n=0,1} (\mathrm{Im}\, T)_n^{\mathrm{cut}}\,, \tag{9}$$

where cut = diffractive, double, single and n is the number of gluons emitted at the vertex. It is important that although the sums (9) satisfy the simple AGK relations (6)–(8), the separate contributions from $n = 0$ and $n = 1$ do not, as we shall see in the following.

3 The triple pomeron contribution in the perturbative QCD

Analysis of reggeized gluon diagrams shows that the amplitude with four final reggeized gluons may be represented in different forms. From the direct study of the triple discontinuity with two, three and four exchanged reggeized gluons one finds an expression which is a sum of the double pomeron exchange and triple pomeron contribution with the so-called diffractive vertex Z [12–14]. However, neither the vertex Z nor the gluon coupling to the external particles in the double pomeron exchange contribution is symmetric in all four reggeized gluons 1, . . . , 4 (they are only symmetric in the pairs 12 and 34). So according to the results of the previous section neither of these two contributions can separately satisfy the AGK rules. However, these two contributions can be transformed in two others, one of which (the ‘reducible’ part) has the form of single pomeron exchange and the other (the ‘irreducible’ part) of a triple pomeron contribution with a different vertex V, symmetric in all the gluons [12]. Remarkably in the high-color limit this vertex coincides with the one introduced by Mueller and Patel in the color dipole model [15], and which represents the pomeron interaction in the hA and AB collisions. Our results show that it is this vertex for the triple pomeron interaction which satisfies the AGK rules. Note that this statement was made rather long ago in connection with the amplitude for the scattering on a single center [10, 12]. Our results generalize it to the scattering on two (or many) centers.

For our purpose we only need the vertex projected onto the incoming pomeron colorless color state:

$$\sum_b \Gamma^{bb}_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2) \equiv \Gamma_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2)\,. \tag{10}$$

This vertex is related to the symmetric Bartels vertex V by the relation

$$g^2\Gamma_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2) = \frac{1}{2}q_1^2q_2^2 V_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2)\,. \tag{11}$$

The vertex V has the following color and momentum structure:

$$\begin{aligned} V_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2) &= \delta_{a_1a_2}\delta_{a_3a_4}V(1,2,3,4) + \delta_{a_1a_3}\delta_{a_2a_4}V(1,3,2,4) \\ &\quad + \delta_{a_1a_4}\delta_{a_3a_2}V(1,4,3,2)\,. \end{aligned} \tag{12}$$

For brevity we suppress the dependence on the momenta of the initial pomeron q_1, q_2 common to all the terms and denote the momenta of the final gluons by their numbers, so 1 means k_1 and so on. The function $V(1,2,3,4)$ is symmetric under the interchanges $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ and $12 \leftrightarrow 34$. The whole expression (12) is obviously completely symmetric in all four gluons.

The explicit expression for the function $V(1,2,3,4)$ is conveniently given in terms of the function $G(k_1,k_2,k_3)$ introduced in [12] and generalized to the non-forward direction in [16]:

$$\begin{aligned} V(1,2,3,4) = \frac{g^2}{2}\big(&G(1,23,4) + G(2,13,4) + G(1,24,3) \\ &+ G(2,14,3) - G(12,3,4) - G(12,4,3) \\ &- G(1,2,34) - G(2,1,34) + G(12,0,34)\big)\,, \end{aligned} \tag{13}$$

where again for brevity we denote $12 = 1 + 2 = k_1 + k_2$ etc. The function G has the form

$$\begin{aligned} G(k_1,k_2,k_3|q_1,q_2) = &-g^2N_cK(k_1,k_2,k_3|q_1,g_2) \\ &- (2\pi)^3\delta^2(q_1-k_1)\,(\omega(2)-\omega(23)) \\ &- (2\pi)^3\delta^2(q_2-k_3)\,(\omega(2)-\omega(12))\,. \end{aligned} \tag{14}$$

Here $\omega(k)$ is the gluon Regge trajectory and the kernel for the transition of two gluons into three K is given by

$$\begin{aligned} K(k_1,k_2,k_3|q_1,q_3) = &\frac{(k_1+k_2+k_3)^2}{q_1^2q_3^2} \\ &+ \frac{k_2^2}{(k_1-q_1)^2(k_3-q_3)^2} \\ &- \frac{(k_1+k_2)^2}{q_1^2(k_3-q_3)^2} - \frac{(k_2+k_3)^2}{q_3^2(k_1-q_1)^2}\,. \end{aligned} \tag{15}$$

It conserves momentum, so $k_1 + k_2 + k_3 = q_1 + q_2$.

Summation over the colors of the final pomerons in the expression for the amplitude (1) transforms $\Gamma_{a_1a_2a_3a_4}(k_1,k_2,k_3,k_4|q_1,q_2)$ into

$$\begin{aligned}\Gamma(k_1,k_2,k_3,k_4|q_1,q_2)\\ &=\left(\frac{1}{N_c^2-1}\right)^2\sum_{a_1,a_3}\Gamma_{a_1a_1a_3a_3}(k_1,k_2,k_3,k_4|q_1,q_2)\\ &=\frac{1}{2}q_1^2q_2^2\left[V(1,2,3,4)\right.\\ &\left.+\frac{1}{N_c^2-1}\left(V(1,3,2,4)+V(1,4,3,2)\right)\right]. \quad (16)\end{aligned}$$

In the high-color limit only the first term remains. The function $F(j_i,t_i)$ in the integrand for the amplitude (1) becomes

$$\begin{aligned}F(j_i,t_i)=\int\frac{\mathrm{d}^2q_1}{(2\pi)^2}\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\Gamma(k_1,k_2,k_3,k_4|q_1,q_2)\\ \times P_1(j_1;k_1,k_2)P_2(j_2;k_3,k_4)P(j_0;q_1,q_2)\,.\end{aligned} \quad (17)$$

Note that the resulting vertex Γ is no more symmetric in all the gluons, which is a consequence of an unsymmetrical projection onto the color space of the final reggeons. Putting in (1) $s_1=s_2=s$, passing to rapidities defined by

$$y=\ln\frac{s}{M^2}\,,\quad Y=\ln\frac{s}{s_0}\,, \quad (18)$$

and substituting the signature factors by their lowest order values, we obtain for the amplitude the standard expression

$$\begin{aligned}T_{3\to3}=-\mathrm{i}s\frac{1}{2\pi}\int\frac{\mathrm{d}^2q_1}{(2\pi)^2}\frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}\Gamma(k_1,k_2,k_3,k_4|q_1,q_2)\\ \times P_1(y;k_1,k_2)P_2(y;k_3,k_4)P(Y-y;q_1,q_2)\,, \quad (19)\end{aligned}$$

where the pomerons in the y-representation are defined as

$$P(y)=\int\frac{\mathrm{d}j}{2\pi\mathrm{i}}\mathrm{e}^{y(j-1)}P(j)\,. \quad (20)$$

This amplitude refers to the case when we have a simple triple pomeron diagram corresponding to scattering on two centers. A more important case is scattering on many centers (heavy nucleus) described by a sum of all fan diagrams made of the BFKL pomerons with their triple interaction. Apart from the driving term, which is an exchange of a single pomeron, it is given by the expression of the same form as (19) in which the final pomerons are substituted by the sums of all fans $\Phi(y,k_1,k_2;b)$ and $\Phi(y,k_3,k_4;b)$ where b is the impact parameter.

The expression (19) can be further simplified taking the limit $N_\mathrm{c}\to\infty$ and passing to the coordinate representation [16]. Here we want only to comment on different parts of the imaginary part of this amplitude corresponding to different cuts. They will generally involve different projections onto the color space of the final pomerons. However, due to the symmetry of the vertex Γ the final result will be the same. For instance in the double cut we find the vertex

$$\left(\frac{1}{N_c^2-1}\right)^2\sum_{a_1,a_3}\Gamma_{a_1a_3a_1a_3}(k_1,k_3,k_2,k_4|q_1,q_2)\,. \quad (21)$$

However, the symmetry allows one to interchange gluons 2 and 3, and (21) becomes identical to (16). So all contributions to the imaginary part will contain the same vertex $\Gamma(k_1,k_2,k_3,k_4|q_1,q_2)$ defined by (16) and so will be given by the same expression (19) with the factor $-\mathrm{i}$ changed to 1, 2 and -4 for the diffractive, double cut and single cut contributions, respectively.

4 Single inclusive cross-section

As mentioned the total contribution to the amplitude for scattering on two centers can be presented as a sum of the amplitude $T_{3\to3}$, studied in the previous section, and the reducible part, which is just a pomeron coupled simultaneously to the two centers. Our central interest will be the inclusive cross-section corresponding to the triple pomeron contribution $T_{3\to3}$. The contribution from the reducible part is simpler and will be briefly discussed later. To derive the single inclusive cross-section for emission of a gluon jet at rapidity y and of the transverse momentum k we have to find it in the intermediate states in the unitarity relation for $T_{3\to3}$. The amplitude $T_{3\to3}$ presented by (1) has three pomeron legs, each having a clear and well-known representation in terms of reggeon diagrams. At rapidity y_V the pomerons are joined by the vertex Γ, whose representation in terms of reggeon diagrams is not possible. The important point is that the vertex is local in rapidity belonging to fixed rapidity y_V, unlike the pomerons which are objects extended in rapidity. From this one concludes that an intermediate real gluon with rapidity $y\gg y_V$ or $y\ll y_V$ can only be present in the pomeron legs. Since the structure of these legs in terms of reggeon diagrams is known, the corresponding inclusive cross-section can be found by just "opening" the BFKL chain. However apart from this pomeron contribution, there also may appear a contribution at exactly y_V from the gluon emission from the vertex itself. These considerations demonstrate that the inclusive cross-sections satisfy the standard reggeization pattern when the observed particles can be extracted either from the reggeons or from their joining vertexes.

"Opening" the BFKL chain is described in the coordinate space by inserting the emission operator [17]

$$V_k(r)=\frac{4\alpha_\mathrm{s}N_c}{k^2}\overleftarrow{\Delta}\mathrm{e}^{\mathrm{i}kr}\overrightarrow{\Delta}\,, \quad (22)$$

that is, substituting the BFKL Green function at rapidity interval Y by

$$G(Y;r_1,r_2)\to\int\mathrm{d}^2rG(Y-y;r_1,r)V_k(r)G(y:r,r_2)\,. \quad (23)$$

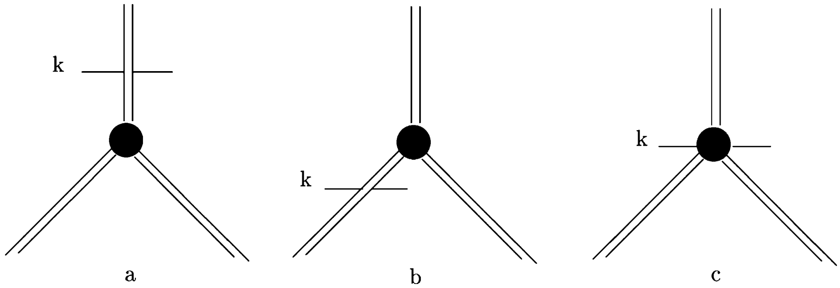

Fig. 4. Pomeron diagrams for the single inclusive cross-section on two centers

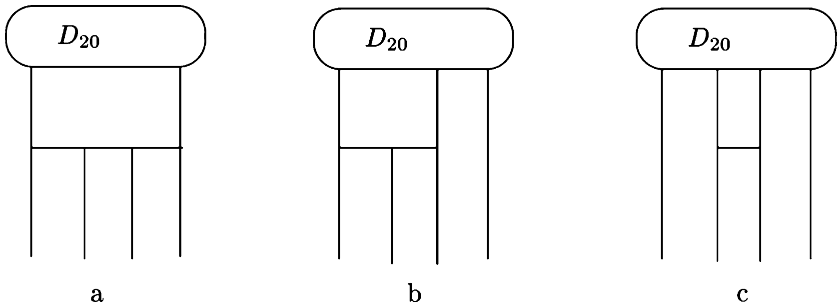

Fig. 5. Transitions from **a** 2 to 4, **b** 3 to 4 and **c** 4 to 4 gluons

In this way we find the single inclusive cross-section at fixed impact parameter b on two centers corresponding to the emission from the upper pomeron in $T_{3\to 3}$ (see Fig. 4a):

$$J^{(P)}(y,k) \equiv \frac{(2\pi)^3\,\mathrm{d}\sigma^{(P)}}{\mathrm{d}y\,\mathrm{d}^2k\,\mathrm{d}^2b} = 2\int \mathrm{d}^2r P(Y-y;r)V_k(r)\Phi^{(2)}(y;r|b)\,. \tag{24}$$

Here $P(Y-y;r)$ is the initial pomeron in the coordinate representation. The function $\Phi^{(2)}(y;r|b)$ is the contribution from the double interaction with the nucleus due to the three-pomeron vertex. In the momentum representation

$$\Phi^{(2)}(y;q_1,q_2|b) = T^2(b)\int \frac{\mathrm{d}^2k_1}{(2\pi)^2}\frac{\mathrm{d}^2k_3}{(2\pi)^2}P_1(y;k_1,k_2) \times P_2(y;k_3,k_4)\Gamma(k_1,k_2,k_3,k_4|q_1,q_2)\,, \tag{25}$$

with $q_1+q_2=0$. The AGK relations (6)–(8) tell us that emissions from lower pomerons in Φ (Fig. 4b) do not give any contribution.

Thus we are left with the emission from within the vertex Γ corresponding to the diagram shown in Fig. 4c. To find this contribution it is evidently enough to study the inclusive cross-section for the case when the three pomerons joined at the vertex are taken in the lowest order: the double gluon exchange. In this case there is no gluon emission from inside the pomerons and all gluons come either from the vertex or from the additional contribution separated from the three-pomeron diagram as the mentioned reducible part in the reggeon diagram technique, or equivalently in the form of Glauber rescattering in the initial state in the evolution equation for the sum of fans Φ. All we have to do is to study in the lowest order all the corresponding diagrams with four gluon legs combined into the final pomerons and locate the observed intermediate gluon in the appropriate cuts. The number of initial gluons, coupled to the quark–antiquark loop may be two, three or four. Correspondingly the diagrams split into three types with transitions $2\to 4$, $3\to 4$ and $4\to 4$ gluons.

Typical diagrams for these three cases are shown in Fig. 5a–c. The upper blob, D_{20}, represents the quark–antiquark loop with two, three or four gluons attached to it in all possible ways conserving the order of gluons 1, 2, 3 and 4. The final gluons are to be understood as parts of the lower pomerons in $T_{3\to 3}$. They are to be combined into pomerons in two different configurations: the diffractive one, in which the pomerons are made of pairs $(1,2)$ and $(3,4)$, and the double cut one with pomerons made of pairs $(1,4)$ and $(2,3)$. Note that the pairing $(1,3)$ and $(2,4)$ is prohibited. It is well known that two ladders can couple to a particle only in the so-called "nested" configuration. This can also be understood from the kinematical situation: the rungs in the pomeron ladders do not depend on the longitudinal variables and so are instantaneous in the longidudinal directions. Diffractive and double cut configuration allow the relative longitudinal distance between the two pomerons to be arbitrary, which translates into the integration of the nuclear density into the profile function. In the configuration $(1,3)$ $(2,4)$ the two pomerons are in fact located at the same longitudinal coordinate, which leads to suppression of this contribution by the inverse of the large nuclear dimension. The diffractive and double cuts are made between the gluons 2 and 3 (see Fig. 3a and b). The single cuts are made in both configurations separating gluons 1 or 4 (Fig. 3c and d). Actual calculations are best performed in the coordinate space (although naturally they are completely equivalent to the calculations in the momentum space customary in the reggeon diagram technique). The vertexes for the transitions $n\to 4$ gluons with

$n = 2, 3, 4$ can be conveniently represented via the emission vertexes reggeon $\to$ reggeon + particle (Lipatov) and reggeon $\to$ m reggeons + particle (Bartels) which may be found in [12]. In the coordinate space emission of a gluon of momentum k at point $\mathbf{z}_1$ from the quark at point $\mathbf{r}_1$ by the Lipatov vertex is described by a factor

$$f_1(r_1) = k_1(r_1) - h_1(r_1), \quad k_1(r_1) = \frac{\mathbf{k}}{k^2}\delta^2(z_1 - r_1),$$
$$h_1(r_1) = \frac{\mathrm{i}}{2\pi}\frac{\mathbf{z}_1 - \mathbf{r}_1}{(z_1 - r_1)^2}. \tag{26}$$

Similarly $f_1(r_2)$ refers to the emission from an antiquark with coordinate $\mathbf{r}_2$, and $f_2^*(r_1)$ and $f_2^*(r_2)$ refer to the conjugate amplitude with the emission point $\mathbf{z}_2$. Emission by the Bartels vertex is described by a factor

$$B_1(r_1, x) = h_1(r_1)\left(\delta^2(x - r_1) - \delta^2(x - z_1)\right), \tag{27}$$

where $\mathbf{x}$ is the coordinate of splitting of the first final gluon. The rest of the final gluons are to be located at $\mathbf{z}_1$.

Following [8] we present the result for this lowest order inclusive cross-section in the form of the integral over the interquark distance in the loop $r_{12} = r_1 - r_2$ and gluon emission points in the direct and conjugate amplitudes:

$$J_0(y, k) \equiv \frac{(2\pi)^3\,\mathrm{d}\sigma_0}{\mathrm{d}y\,\mathrm{d}^2k\,\mathrm{d}^2b}$$
$$= g^2 N_c T^2(b)\int \mathrm{d}^2r_{12}\,\mathrm{d}^2z_1\,\mathrm{d}^2z_2\mathrm{e}^{-\mathrm{i}kz}$$
$$\times I(r_1, r_2, z_1, z_2)\nabla^4 P^{(0)}(r_{12}). \tag{28}$$

Here $P^{(0)}(r)$ is the upper pomeron in the lowest order. Application of ∇^4 removes its legs and leaves only the quark–antiquark loop in the coordinate representation $D_{20}(r)$. The integrand I is a sum of contributions from the diffractive, double and single cuts:

$$I = I^{\mathrm{diff}} + I^{\mathrm{double}} + I^{\mathrm{single}}. \tag{29}$$

Each of them is just the sum of the contributions from the appropriately cut diagrams with the statistical weight factors 2, 4 and -4 for the diffractive, double and single cuts respectively.

To write down the contributions we use the shorthand notation for the lower pomerons (we consider them identical):

$$X \equiv P(\mathbf{z}_1 - \mathbf{r}_1), \quad Y \equiv P(\mathbf{z}_1 - \mathbf{r}_2), \quad U \equiv P(\mathbf{z}_2 - \mathbf{r}_1),$$
$$V \equiv P(\mathbf{z}_2 - \mathbf{r}_2), \quad Z \equiv P(\mathbf{z}_1 - \mathbf{z}_2), \quad R \equiv P(\mathbf{r}_1 - \mathbf{r}_2). \tag{30}$$

As mentioned the pomerons are to be taken in the lowest order (two-gluon exchange) and so do not depend on the rapidity.

Simple although somewhat tedious calculations give the following results for the contributions from the diffractive, double and single cuts:

$$I^{\mathrm{dif}} = 2(h_1(r_1) - h_1(r_2))(h_2^*(r_1) - h_2^*(r_2))$$
$$\times (X + Y - R)(U + V - R), \tag{31}$$

$$I^{\mathrm{double}} = (h_1(r_1) - h_1(r_2))(h_2^*(r_1) - h_2^*(r_2))$$
$$\times\left[4Z^2 + 2(XV + YU) - 3Z(X + Y + U + V)\right]$$
$$+ h_1(r_1)h_2^*(r_1)\,[U(R + 3V - U) + X(R + 3Y - X)]$$
$$+ h_1(r_2)h_2^*(r_2)\,[V(R + 3U - V) + Y(R + 3X - Y)]$$
$$- h_1(r_1)h_2^*(r_2)\,[U(R + 3V - U) + Y(R + 3X - Y)]$$
$$- h_1(r_2)h_2^*(r_1)\,[V(R + 3U - V) + X(R + 3Y - X)], \tag{32}$$

$$I^{\mathrm{single}} = (h_1(r_1) - h_1(r_2))(h_2^*(r_1) - h_2^*(r_2))$$
$$\times\{3Z\,[X + Y + U + V]$$
$$-2XU - 2YV - 4XV - 4YU\}$$
$$+ (h_1(r_1)U - h_1(r_2)V)(h_2^*(r_1) - h_2^*(r_2))$$
$$\times[-2R - 3(U + V)]$$
$$+ (h_1(r_1) - h_1(r_2))(h_2^*(r_1)X - h_2^*(r_2)Y)$$
$$\times[-2R - 3(X + Y)]$$
$$+ h_1(r_1)h_2^*(r_1)R\,(2V + 3U + 2Y + 3X)$$
$$+ h_1(r_2)h_2^*(r_2)R\,(2U + 3V + 2X + 3Y)$$
$$- h_1(r_1)h_2^*(r_2)R\,(2V + 3U + 2X + 3Y)$$
$$- h_1(r_2)h_2^*(r_1)R\,(2U + 3V + 2Y + 3X)$$
$$- 2R^2\,(h_1(r_1) + h_1(r_2))\,(h_2^*(r_1) + h_2^*(r_2)). \tag{33}$$

Remarkably in the sum of these three contributions nearly all the terms cancel and one gets a comparatively simple expression:

$$I = 4h_1(r_1)h_2^*(r_1)\left(Z^2 - X^2 - U^2\right)$$
$$+ 4h_1(r_2)h_2^*(r_2)\left(Z^2 - Y^2 - V^2\right)$$
$$- 4h_1(r_1)h_2^*(r_2)\left(R^2 + Z^2 - Y^2 - U^2\right)$$
$$- 4h_1(r_2)h_2^*(r_1)\left(R^2 + Z^2 - X^2 - V^2\right). \tag{34}$$

It agrees with the expression found in the Glauber approach in [8]

As mentioned before, to find the contribution from the vertex one has to subtract from this expression the term which comes from the reducible part. In the coordinate representation this part comes from the contribution of the double scattering in the Glauber expression for the initial state:

$$T^{\mathrm{red}} = \int \mathrm{d}^2r P_1^{(0)}(r)P_2^{(0)}(r)P(Y, r), \tag{35}$$

where the upper pomeron is a developed one, but the two lower ones are to be taken in the lowest order. The contribution to the inclusive cross-section is obtained by opening the upper pomeron (and multiplying the result by 2 for the twice imaginary part). The found inclusive cross-section can again be represented in the form (28) with an integrand

$$I^{\mathrm{red}} = \frac{1}{2}I. \tag{36}$$

This brings us to the final contribution from the vertex, which has the form (28) with an integrand

$$I^{\text{vertex}} = \frac{1}{2} I = \frac{1}{2}\left(I^{\text{diff}} + I^{\text{double}} + I^{\text{single}}\right) . \tag{37}$$

The three terms in (37) may be related to the expressions for the vertex cut in different ways: diffractive, double cut or single cut. We admit that such an interpretation has a somewhat heuristic character. For the single inclusive cross-section it is not needed: only a sum of these three terms appears in it. However, the study of the double inclusive cross-section, as we shall see in the following, requires knowledge of the vertex separately for different cuttings. Then we shall use the interpretation following from (37).

The simple form of I allows one to do the integrations over z_1 and z_2 and present the inclusive cross-section from the vertex in a simpler form similar to (28) [8]:

$$J_0^{(V)}(y,k) = -T^2(b)\int \mathrm{d}^2 r P^{(0)}(r) V_k(r) [P^{(0)}(r)]^2 . \tag{38}$$

This contribution has been found for the case when all pomerons are taken in the lowest order, without evolution. However from the structure of the triple pomeron diagram it immediately follows that evolution just restores all orders for all the three pomerons, so that the inclusive cross-section coming from emission from the vertex is given by the same expression (38) with the pomerons taken fully evolved, the upper one up to $Y-y$ and the two lower ones up to y. We have

$$J^{(V)}(y,k) \equiv \frac{(2\pi)^3 \mathrm{d}\sigma^{(V)}}{\mathrm{d}y\, \mathrm{d}^2 k\, \mathrm{d}^2 b} = -\int \mathrm{d}^2 r P(Y-y;r) V_k(r) [\Phi^{(1)}(y;r|b)]^2 , \tag{39}$$

where $\Phi^{(1)}$ corresponds to a single interaction with the nucleus:

$$\Phi^{(1)}(y;r|b) = P(y;r) T(b) . \tag{40}$$

Passing to scattering on many centers we have to take into account that due to the AGK cancellations ony the contributions from the uppermost pomeron and vertex remain. So to obtain the inclusive cross-section we have only to appropriately change the lower legs in the contributions from the single and double scattering. In this way we get the final inclusive cross-section as

$$J(y,k) \equiv \frac{(2\pi)^3 \mathrm{d}\sigma}{\mathrm{d}y\, \mathrm{d}^2 k\, \mathrm{d}^2 b} = \int \mathrm{d}^2 r_1 \mathrm{d}^2 r P(Y-y;r) V_k(r) \times \left(2\Phi(y;r|b) - \Phi^2(y;r|b)\right) . \tag{41}$$

The second term corresponds to emission from the vertex and agrees with the result of [8].

Note finally that integrating the contributions found from different cuts over k we obtain the total imaginary parts of the amplitude due to emission of a gluon from the vertex, divided by s and so coinciding with the corresponding cross-sections σ_1:

$$\sigma_1^{\text{dif}} = 4\alpha_s (h_1(r_1) - h_1(r_2))^2 (X+Y-R)^2 , \tag{42}$$

$$\begin{aligned}\sigma_1^{\text{double}} = 2\alpha_s \{ & 2h_1^2(r_1)(XR + 5XY - X^2) \\ & + 2h_1^2(r_2)(YR + 5XY - Y^2) \\ & - 2h_1(r_1)h_1(r_2) \\ & \times \left(R(X+Y) + 10XY - X^2 - Y^2\right)\} ,\end{aligned} \tag{43}$$

$$\begin{aligned}\sigma_1^{\text{single}} = 2\alpha_s \{ & 2h_1^2(r_1)\left(R(X+2Y) - 7XY - 4X^2 - Y^2\right) \\ & + 2h_1^2(r_2)\left(R(2X+Y) - 7XY - X^2 - 4Y^2\right) \\ & + 2h_1(r_1)h_2(r_2) \\ & \times \left(3R(X+Y) + 14XY + 5X^2 + 5Y^2\right)\} .\end{aligned} \tag{44}$$

One observes that they do not satisfy the AGK relations (6)–(8). Only summed with the cross-sections without emission of a gluon from the vertex they do satisfy these relations.

5 Evolution equation for the inclusive cross-section on two centers

Evolution of the inclusive cross-section with rapidities follows directly from the representation (28) and the evolution equations for $P(Y-y;r)$ and $\Phi(y;r|b)$. At a fixed number of centers however the structure of the inclusive cross-section allows one to construct an evolution equation directly for it, as an equation which describes evolution of the lower legs. We shall limit ourselves to the inclusive cross-section on two centers (two lower pomerons).

As mentioned, the amplitude $T_{3\to 3}$ itself can be split into a reducible and irreducible part. Separating from the irreducible part the nuclear sources with the final gluon propagators one obtains the irreducible four-gluon amplitude $D_4^{(I)}$ studied in [12, 18]. It satisfies an equation obvious from the representation (1), which can be written symbolically as

$$\left(\frac{\partial}{\partial y} + H_4\right) D_4^{(I)}(y) = \Gamma P(y) . \tag{45}$$

Here H_4 is the Hamiltonian for four reggeized gluons which form the final pomerons. $P(y)$ is the upper pomeron. Γ is the three-pomeron vertex as an operator acting from the space of the two initial gluons into the space of the four final gluons. In the limit $N_c \to \infty$ the Hamiltonian H_4 contains only the BFKL interactions between the gluons inside each of the two pomerons. The solution of (45) can be achieved by applying to the right-hand side the Green function for four reggeized gluons:

$$D_4^{(I)}(y) = \int \mathrm{d}y' G_4(y-y') \Gamma P(y') . \tag{46}$$

In the limit $N_c \to \infty$ G_4 is just a product of two independent BFKL Green functions for the two pomerons. Attaching the sources to (46) one restores the representation (1).

Passing to the inclusive cross-section (41), for two scattering centers we can introduce a similar amplitude $D_4^{(I)}(Y,y|k)$, which is obtained by separating from the cross-section the two nuclear sources and changing the direction of the evolution, so that the nucleus is at rapidity Y, the splitting occurs at y, and the projectile is at zero rapidity. From (41) we find

$$D_4^{(I)}(Y,y|k) = \int \mathrm{d}y' G_4(Y-y')\Gamma P(y',y|k) - G_4(Y-y)V_k P(y)\,, \tag{47}$$

where $P(Y,y|k)$ is an opened pomeron:

$$P(Y,y|k) = G(Y-y)V_k P(y)\,. \tag{48}$$

Note that here V_k acts exclusively in the two-gluon space whereas in (47) it acts from the two-gluon into four-gluon space. Obviously $D_4^{(I)}(Y,y|k)$ satisfies the equation

$$\left(\frac{\partial}{\partial Y} + H_4\right) D_4^{(I)}(Y,y|k)) = \Gamma P(Y,y|k) - \delta(Y-y)V_k P(y). \tag{49}$$

Indeed applying the Green function G_4 to the right-hand side one obtains (47). Comparing (49) with (45) one observes that the equations are actually quite similar at $Y \neq y$. The role of the second term describing emission from the vertex is only to supply the initial condition for the evolution at $Y = y$:

$$D_4^{(I)}(y,y|k)) = -G(0)V_k P(y)\,. \tag{50}$$

6 Double inclusive cross-section

Now we study the double inclusive cross-section for emission of two jets with rapidities and transverse momenta y_1, k and y_2, l. We assume $y_1 > y_2$ (and in fact in the BFKL kinematics $y_1 \gg y_2$). Both gluons may come from within the pomerons. The AGK rules tell us that the only contributions of this sort which remain are from either two emissions from the upper pomeron or from emissions from the both lower pomerons immediately after the first splitting shown in Fig. 6a and b. These are the standard AGK contributions.

Now we pass to contributions which involve emission from the vertex. Clearly in the lowest order there cannot occur two emissions from the vertex at the same rapidity. According to the AGK rules we are left with two cases: either the faster gluon (rapidity y_1) is emitted from the uppermost pomeron and the slower one from the first splitting vertex (Fig. 6c) or the faster gluon is emitted from the uppermost vertex and the slower one from one of the lower pomerons immediately after the first splitting (Fig. 6d).

The first case is simple. In this case the lower pomerons can be cut in any way and the contribution from the vertex includes all three terms corresponding to diffractive,

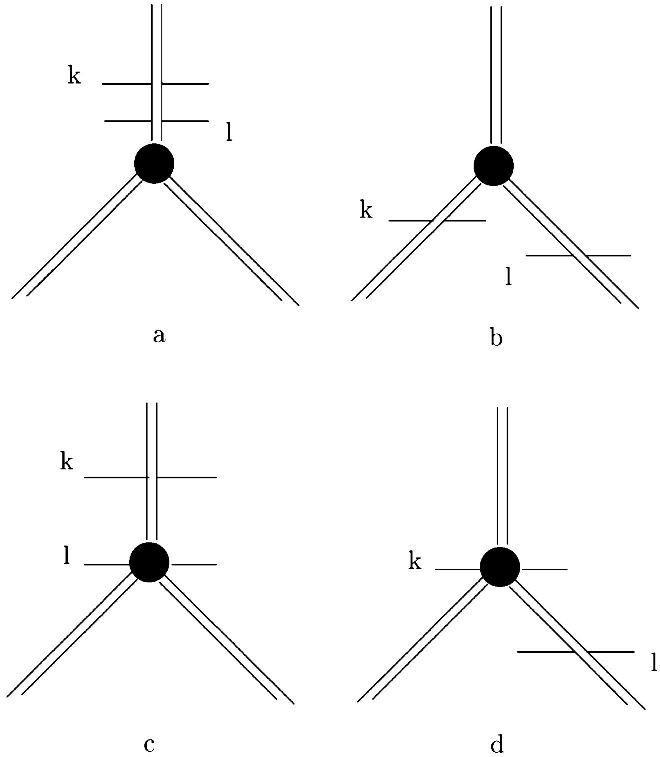

Fig. 6. Pomeron diagrams for the double inclusive cross-section on two centers

double and single cuts, which combine into the final expression (38) for emission from the vertex. So the double inclusive cross-section in this case is obtained from (38) by just additionally 'opening' the upper pomeron:

$$J^{(1)}(y_1,k;y_2,5r'l) \equiv \frac{(2\pi)^6\,\mathrm{d}\sigma}{\mathrm{d}y_1\,\mathrm{d}^2k\,\mathrm{d}y_2\,\mathrm{d}^2l\,\mathrm{d}^2b} = -\int \mathrm{d}^2r\,\mathrm{d}^2r' P(Y-y_1;r)V_k(r) \times G(y_1-y_2;r,r')V_l(r')\Phi^2(y_2;r'|b)\,. \tag{51}$$

The second case is more complicated. Now the lower pomeron which emits the gluon must be cut. This excludes the diffractive part of the contribution to the emission from the vertex and instead of the simple expression (38) we have to use the sum of only double and single cuts for the vertex emission. The integrand in (28) as a result is more complicated:

$$I_1 = (1/2)(I^{\mathrm{double}} + I^{\mathrm{single}}) = (1/2)(I - I^{\mathrm{dif}})\,. \tag{52}$$

By shifting variables z_1 and z_2 one can present the contribution to emission from the vertex from only the diffractive cut in the form

$$J^{\mathrm{dif}}(y,k) = T^2(b)\int \mathrm{d}^2r\,\mathrm{d}^2r_1\,\mathrm{d}^2r_2 P_1(y,r_1)P_2(y,r_2) \times P(Y-y,r)\Gamma_k^{\mathrm{dif}}(r_1,r_2|r)\,. \tag{53}$$

The emission vertex $\Gamma_k^{\mathrm{dif}}(r_1,r_2|r)$ can be presented via the kernel K, see (15), in which the momenta are substituted for by coordinates (not the Fourier transform!):

$$\Gamma_k^{\mathrm{dif}}(r_1, r_2|r) = g^2 N_c \mathrm{e}^{-\mathrm{i}k(r_1-r_2)} \{K(r, r_1+r_2-2r, r|r_1, r_2) - \mathrm{e}^{-\mathrm{i}kr} K(r, r_1+r_2, -r|r_1, r_2) - \delta^2(r-r_1)\int \mathrm{d}^2 r_2' K(r, r_1+r_2'-2r, r|r_1, r_2') - \delta^2(r-r_2)\int \mathrm{d}^2 r_1' K(r, r_1', r_2-2r, r|r_1', r_2) + \frac{1}{2}\delta^2(r-r_1)\delta^2(r-r_2) \times \int \mathrm{d}^2 r_1' \, \mathrm{d}^2 r_2' K(r, r_1'+r_2'-2r, r|r_1', r_2')\} . \quad (54)$$

The double inclusive cross-section with emission of the fastest gluon jet from the vertex and the other from the lower pomeron will be given by 'opening' in (38) the lower pomeron and subtracting from the contribution from the vertex the diffractive cut part:

$$J^{(2)}(y_1, k; y_2, l) = -\int \mathrm{d}^2 r P(Y-y_1, r) V_k(r) \Phi(y; r|b) \times G(y_1-y_2; r, r') V_l(r') \Phi(y_2; r'|b) - \int \mathrm{d}^2 r \, \mathrm{d}^2 r_1 \, \mathrm{d}^2 r_1' \, \mathrm{d}^2 r_2 P(Y-y_1, r) \times \Gamma_k^{\mathrm{dif}}(r_1, r_2, r) \Phi(y; r_2|b) G(y_1-y_2; r_1, r_1') \times V_l(r_1') \Phi(y_2; r_1'|b) + (r_1 \leftrightarrow r_2) . \quad (55)$$

7 Conclusions

We have established that the AGK rules can only be satisfied if the triple pomeron vertex is a fully symmetric function in all four reggeized gluons which form the two outgoing pomerons. This selects the symmetric Bartels vertex V as a vertex for the triple pomeron amplitude obeying the AGK rules. The total amplitude for the scattering on two centers thus splits into this triple pomeron part and a single pomeron exchange one, both satisfying the AGK rules in their own way (the latter trivially). The unitarity content for the triple pomeron amplitude allows one to determine the contribution from gluon emissions from inside the pomeron in a straightforward manner. The contribution from the vertex emission can be derived neglecting the evolution of the pomerons and summing the relevant reggeized gluon diagrams. For the single inclusive cross-section this leads to the result obtained in [8] from the dipole picture. The 'opened' triple pomeron vertex describing gluon emission from it is found to depend on the nature of the cut through it, which point was mentioned when the AGK rules were derived in [7]. As a result, in the double inclusive cross-section with the slower gluon emitted after the splitting the cut opened vertex is different and more complicated than in the single inclusive cross-section, since the contribution from the diffractive cut has to be dropped. Still the fundamental AGK properties are found to be valid also for the double inclusive cross-section, so that the contribution from the upper pomeron and the one after the first branching claimed in [11] does not appear.

As a byproduct we constructed an evolution equation for the single inclusive cross-section on two centers as a function of the overall rapidity. This equation may be helpful in generalizing to the case of finite N_c, when the final pomerons interact between themselves.

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